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Pair Production and Correlated Decay of Heavy Majorana Neutrinos in e^+e^- Collisions

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Abstract

We consider the process $e^+e^- \to N_1N_2$, where N_1 and N_2 are heavy Majorana particles, with relative CP given by $\eta_{CP} = +1$ or -1, decaying subsequently via $N_1, N_2 \to W^\pm e^\mp$. We derive the energy and angle correlation of the dilepton final state, both for like-sign ($e^\mp e^\mp$) and unlike-sign (e^-e^+) configurations. Interesting differences are found between the cases $\eta_{CP} = +1$ and -1. The characteristics of unlike-sign e^+e^- dileptons originating from a Majorana pair N_1N_2 are contrasted with those arising from the reaction $e^+e^- \to N\bar{N} \to W^+e^-W^-e^+$, where $N\bar{N}$ is a Dirac particle-antiparticle pair.

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I. INTRODUCTION

In an interesting paper [1], Kogo and Tsai have analysed the reaction $e^+e^- \to N_1N_2$, where $N_{1,2}$ are heavy Majorana neutrinos, and compared the cases where the relative CP of N_1 and N_2 is $\eta_{CP} = +1$ and -1. It was found that the two cases differ in threshold behaviour, in angular distribution, and in the dependence on the spin-directions of N_1 and N_2. A comparison was also made between the Majorana process and the Dirac process $e^+e^- \to N\bar{N}$, where $N\bar{N}$ is a Dirac particle-antiparticle pair. A related analysis was carried out in Ref. [2]. (The contrast between Majorana neutrinos and Dirac neutrinos has been the subject of several other papers (e.g. [3–6]) and monographs ([7,8]))

In the present paper, we examine how the differences between the cases $\eta_{CP} = +1$ and -1 propagate to the decay products of N_1 and N_2, assuming the decays to take place via $N_{1,2} \to W^\pm e^\mp$. We focus on the like-sign lepton pair created in the reaction chain $e^+e^- \to N_1N_2 \to W^+e^-W^+e^-$, which is a characteristic signature of Majorana pair production. We derive, in particular, the correlation in the energies of the e^-e^- pair, and in their angles relative to the e^+e^- axis. Interesting differences are found between the cases $\eta_{CP} = +1$ and -1. We also examine the behaviour of the unlike-sign dileptons e^+e^-, comparing the Majorana cases with dileptons created in the production and decay of a Dirac $N\bar{N}$ pair, i.e. $e^+e^- \to N\bar{N} \to W^+e^-W^-e^+$.

II. CHARACTERISTICS OF THE REACTION $e^+e^- \to N_1N_2$

The analysis of Ref. [1] was carried out in the context of the simple production mechanism for $e^+e^- \to N_1N_2$ shown in Fig. 1, and we begin by recapitulating the essential results. The interaction Lagrangian is taken to be

$$\begin{aligned}\mathcal{L}_1(x) \;=\; -\frac{g}{2\,cos\,\theta_W} \Big[& \bar{e}\,(x)\,\gamma_\mu\,(c_V - c_A\gamma_5)\;e\,(x) \\ & + \alpha_N\;\bar{N}_1(x)\,\gamma_\mu\,\frac{1}{2}(1-\gamma_5)\;N_2(x) \\ & + \alpha_N\;\bar{N}_2(x)\,\gamma_\mu\,\frac{1}{2}(1-\gamma_5)\;N_1(x)\;\Big]\;Z^\mu(x)\;\;, \qquad (2.1)\end{aligned}$$

where c_V, c_A and α_N may be regarded as real phenomenological parameters. (For the standard Z-boson, $c_V = -1/2 + 2\sin^2\theta_W$, $c_A = -1/2$). The matrix element for Majorana neutrinos (with momenta and spins as indicated in Fig. 1) is

$$\begin{aligned} \mathcal{M}_m \;=\; -i\alpha_N \, (\frac{g}{2\,cos\,\theta_W})^2 \; j^e_\mu \;\; \Delta_Z^{\mu\nu} \Big[\; & \bar{u}_{t_1}(q_1)\,\gamma_\nu\,\frac{1}{2}(1-\gamma_5)\,v_{t_2}(q_2)\,\lambda_2 \\ & - \bar{u}_{t_2}(q_2)\,\gamma_\nu\,\frac{1}{2}(1-\gamma_5)\,v_{t_1}(q_1)\,\lambda_1 \Big] \,, \end{aligned} \tag{2.2}$$

where

$$j^e_\mu \;=\; \bar{v}_{s_2}(p_2)\,\gamma_\mu\,(c_V - c_A\gamma_5)\,u_{s_1}(p_1) \tag{2.3}$$

and

$$\Delta_Z^{\mu\nu} \;=\; \frac{g^{\mu\nu} \;-\; q^\mu q^\nu/m_Z^2}{q^2 \;-\; m_Z^2 \;+ i\, m_Z\,\Gamma_Z} \quad . \tag{2.4}$$

Assuming CP-invariance the factors λ_1, λ_2 in Eq. (2.2) are such that $\lambda_1\lambda_2^* = +1(-1)$ when N_1 and N_2 have the same (opposite) CP-parity [9]. Rewriting the second term in Eq. (2.2) as

$$\bar{u}_{t_2}(q_2)\,\gamma_\nu\,\frac{1}{2}(1-\gamma_5)\,v_{t_1}(q_1) \;=\; \bar{u}_{t_1}(q_1)\,\gamma_\nu\,\frac{1}{2}(1+\gamma_5)\,v_{t_2}(q_2) \quad , \tag{2.5}$$

we observe that the current of the Majorana neutrinos is pure axial vector when N_1 and N_2 have the same CP-parity ($\eta_{CP} = \lambda_1\lambda_2^* = +1$), and pure vector when they have opposite CP ($\eta_{CP} = \lambda_1\lambda_2^* = -1$). In comparison, the matrix element for the Dirac process $e^+e^- \to N\bar{N}$ is

$$\mathcal{M}_d \;=\; -i\alpha_N \, (\frac{g}{2\,cos\,\theta_W})^2 \, j^e_\mu \, \Delta_Z^{\mu\nu} \, \bar{u}_{t_1}(q_1)\,\gamma_\nu\,\frac{1}{2}(1-\gamma_5)\,v_{t_2}(q_2) \quad . \tag{2.6}$$

The differential cross section for $e^+e^- \to N_1N_2$, for general masses m_1 and m_2, and for arbitrary polarizations $\vec{n}$ and $\vec{n}'$ of the two neutrinos is given in the Appendix. In Sec. 5 we compare our formulas with those of Ref. [1], and with special cases treated in other papers. Here we specialise to the case $m_1 = m_2 = m_N$, for which the cross-section ($d\sigma/d\Omega$) in the cases $\eta_{CP} = +1$ and -1 is

$$(\frac{d\sigma}{d\Omega})_+ = \frac{1}{2}\sigma_0\ \beta^3 \left\{ f_1\ [(n_y n'_y - n_x n'_x)S^2 + (1+n_z n'_z)(1+C^2)]\ -\ f_2\ \ 2(n_z+n'_z)C \right\}\ , \qquad (2.7)$$

$$(\frac{d\sigma}{d\Omega})_- = \sigma_0\ \beta \left\{ f_1\ [2-\beta^2+C^2\beta^2+n_z n'_z(\beta^2+C^2(1/\gamma^2+1))+n_x n'_x S^2(1/\gamma^2+1) \right.$$
$$\left. -n_y n'_y S^2\beta^2 + (n_x n'_z + n'_x n_z)2SC/\gamma^2]\ -\ f_2\ [2(n_x+n'_x)S/\gamma^2 + 2(n_z+n'_z)C] \right\}. \quad (2.8)$$

For comparison, the differential cross section of the Dirac process $e^+e^- \to N\bar{N}$ is

$$(\frac{d\sigma}{d\Omega})_d = \frac{1}{2}\sigma_0\ \beta \left\{ f_1\ [(1+C^2\beta^2) - (n_z+n'_z)\beta(1+C^2) \right. \qquad (2.9)$$
$$-(n_x+n'_x)SC\beta/\gamma + n_z n'_z(C^2+\beta^2) + (n_x n'_z + n_z n'_x)SC/\gamma + n_x n'_x S^2/\gamma^2]\ +$$
$$\left. f_2\ [2C\beta - (n_z+n'_z)C(1+\beta^2) - (n_x+n'_x)S/\gamma + 2n_z n'_z C\beta + (n_x n'_z + n'_x n_z)S\beta/\gamma] \right\}.$$

The symbols in Eqs. (2.7)–(2.9) are defined as follows:

$$\sigma_0 = \frac{G_F^2\alpha_N^2}{512\pi^2}\left|\frac{m_Z^2}{s-m_Z^2+im_z\Gamma_Z}\right|^2 s \quad , \quad \beta = (1-4m_N^2/s)^{1/2} \quad , \quad \gamma = (1-\beta^2)^{-1/2}$$
$$(2.10)$$
$$C = \cos\theta \quad , \quad S = \sin\theta \quad , \quad f_1 = 2\ (c_V^2 + c_A^2) \quad , \quad f_2 = 4\ c_V c_A \quad ,$$

θ being the scattering angle of N_1 (or N) with respect to the initial e^- direction. The co-ordinate axes are defined so that the momentum- and spin-vectors of N_1 and N_2 in the e^+e^- c.m. frame have the components

$$q_1^\mu = (\gamma m, 0, 0, \gamma\beta m)\ \ ,\ \ t_1^\mu = (\gamma\beta n_z, n_x, n_y, \gamma n_z)\ \ , \qquad (2.11)$$
$$q_2^\mu = (\gamma m, 0, 0, -\gamma\beta m)\ \ ,\ \ t_2^\mu = (-\gamma\beta n'_z, n'_x, n'_y, \gamma n'_z)\ \ .$$

Inspection of Eqs. (2.7)–(2.9) reveals several interesting features:

(a) The Majorana cases '+' and '−' have different dependence on the spin-vectors $\vec{n}$ and $\vec{n}'$, and different angular distributions, even after the spins $\vec{n}$ and $\vec{n}'$ are summed over. These differences stem from the fact that the matrix element $\mathcal{M}_m$ in Eq. (2.2) effectively involves an axial vector current $\bar{N}_1\gamma_\mu\gamma_5 N_2$ when $\lambda_1\lambda_2^* = +1$ and a vector current $\bar{N}_1\gamma_\mu N_2$ when $\lambda_1\lambda_2^* = -1$.

(b) The Majorana cases '+' and '−' differ from the Dirac case 'd', in which the current of the $N\bar{N}$-pair has a V–A structure $\bar{N}\gamma_\mu \frac{1}{2}(1-\gamma_5)N$. This difference persists even if the spins of the heavy neutrinos are summed over, in which case

$$\begin{aligned}
\sum_{\vec{n},\vec{n}'}(\frac{d\sigma}{d\Omega})_+ &= 2\;\sigma_0\;\beta^3\;\Big[\;f_1(1+C^2)\;\Big]\;\;, \\
\sum_{\vec{n},\vec{n}'}(\frac{d\sigma}{d\Omega})_- &= 4\;\sigma_0\;\beta\;\Big[\;f_1(2-\beta^2+C^2\beta^2)\;\Big]\;\;, \\
\sum_{\vec{n},\vec{n}'}(\frac{d\sigma}{d\Omega})_d &= 2\;\sigma_0\;\beta\;\Big[\;f_1(1+C^2\beta^2)+f_2(2C\beta)\;\Big]\;\;.
\end{aligned} \tag{2.12}$$

Whereas the spin-averaged Majorana cross sections are forward-backward symmetric, the Dirac process has a term linear in $\cos\theta$, with a coefficient proportional to $f_2 = 4\,c_V c_A$. Eq. (2.12) also shows that the threshold behaviour is β^3, β and β for the cases '+', '−' and 'd' respectively. In the asymptotic limit $\beta \to 1$ the Majorana cases '+' and '−' have the same angular distribution $(1+C^2)$, distinct from that of the Dirac process.

(c) In the high energy limit $\beta \to 1$, the Dirac process $e^+e^- \to N\bar{N}$ has a spin-dependence given by

$$(\frac{d\sigma}{d\Omega})_d = \frac{1}{2}\sigma_0\;\beta\;\Big[\;1-(n_z+n_z')+n_z n_z'\;\Big]\Big[\;f_1\;(\;1+C^2\;)\;+\;2\;f_2\;C\;\Big]\;\;. \tag{2.13}$$

The fact that only the longitudinal components (n_z and n_z') of the N, $\bar{N}$ spins appear in this expression is consistent with the expectation that relativistic Dirac neutrinos are eigenstates of helicity. The fact that the cross section (2.13) vanishes when $n_z = -1$, $n_z' = +1$ confirms the expectation that for a V–A current the N and $\bar{N}$ are produced in left-handed and right-handed states, respectively. By comparison, the Majorana processes $e^+e^- \to N_1 N_2$, for $\eta_{CP} = +1$ and -1, have the high energy behaviour $(\beta \to 1)$

$$(\frac{d\sigma}{d\Omega})_+ = \frac{1}{2}\sigma_0\;\Big\{\;f_1\;[(1+C^2)(1+n_z n_z')+S^2(n_y n_y'-n_x n_x')]\;-\;2f_2 C\;(n_z+n_z')\;\Big\}\;, \tag{2.14}$$

$$(\frac{d\sigma}{d\Omega})_- = \frac{1}{2}\sigma_0 \left\{ f_1 \; [(1+C^2)(1+n_z n_z') + S^2(n_x n_x' - n_y n_y')] \; - \; 2 f_2 C \; (n_z + n_z') \right\} . \quad (2.15)$$

Contrary to the Dirac case, the Majorana reactions have an explicit dependence on n_x, n_y and n_x', n_y', reflecting the fact that a relativistic Majorana particle with $m_N \neq 0$ is not necessarily an eigenstate of helicity, and can have a spin pointing in an arbitrary direction. The Majorana cases '+' and '−' differ in the sign of the term proportional to S^2, which contains the transverse (x- and y-) components of the neutrino spins. It is with the purpose of exposing the subtle differences in the spin state of the $N_1 N_2$ and $N\bar{N}$ systems that we investigate in the following sections the dilepton final state created by the decays of the heavy neutrinos via $N_{1,2} \to W^{\pm} e^{\mp}$ and $N(\bar{N}) \to W^+ e^- (W^- e^+)$.

III. LIKE-SIGN DILEPTONS: THE REACTION $e^+e^- \to N_1 N_2 \to W^+W^+e^-e^-$

As seen in the preceding, the spin state and the angular distribution of the Majorana pair produced in $e^+e^- \to N_1 N_2$ depends on the relative CP-parity η_{CP} of the two particles. We wish to see how these differences manifest themselves in the decay products of N_1 and N_2. To this end, we assume that $m_N > m_W$, and that the simplest decay mechanism is $N_{1,2} \to W^{\mp} e^{\pm}$. In particular, the reaction sequence $e^+e^- \to N_1 N_2 \to W^+W^+e^-e^-$ leads to the appearance of two like-sign leptons in the final state, an unmistakable signature of Majorana pair production. (For the purpose of this paper we assume that the W-bosons decay into quark jets, thus avoiding the complications of final states with 3 or 4 charged leptons.)

We have calculated the amplitude of the process $e^+e^- \to N_1 N_2 \to W^+W^+e^-e^-$, depicted in Fig. 2, assuming a decay interaction ($\alpha_N^{'}$ and $\alpha_N^{''}$ beeing real parameters)

$$\begin{aligned}
\mathcal{L}_2(x) \; = \; - \frac{g}{\sqrt{2}} \Big[\; & \alpha_N^{'} \; \bar{e}\,(x) \; \gamma_\mu \; \frac{1}{2}(1-\gamma_5) \; N_1(x) \; W^{\mu -}(x) \\
& + \alpha_N^{'} \; \bar{N}_1(x) \; \gamma_\mu \; \frac{1}{2}(1-\gamma_5) \; e\,(x) \; W^{\mu +}(x) \\
& + \alpha_N^{''} \; \bar{e}\,(x) \; \gamma_\mu \; \frac{1}{2}(1-\gamma_5) \; N_2(x) \; W^{\mu -}(x)
\end{aligned}$$

$$+ \, \alpha_N'' \; \bar{N}_2(x) \; \gamma_\mu \; \frac{1}{2}(1-\gamma_5) \; e \; (x) \; W^{\mu +}(x) \quad \Big] \; . \tag{3.1}$$

This amplitude has the form (see Appendix for details)

$$\begin{aligned}
\mathcal{M} = iA \;\; j^e_\mu \;\; \Delta_Z^{\mu\nu} & \frac{1}{q_1^2 - m_1^2 + im_1\Gamma_1} \cdot \frac{1}{q_2^2 - m_2^2 + im_2\Gamma_2} \cdot \\
& \Big[\; m_2 \lambda_2 \bar{u}_{t_1}(k_1) \; \gamma_\rho \; \not{q}_1 \; \gamma_\nu \; \gamma_\sigma \; \frac{1}{2}(1+\gamma_5) \; v_{t_2}(k_2) \\
& - m_1 \lambda_1 \bar{u}_{t_2}(k_2) \; \gamma_\sigma \; \not{q}_2 \; \gamma_\nu \; \gamma_\rho \; \frac{1}{2}(1+\gamma_5) \; v_{t_1}(k_1) \Big] \; \epsilon^{*\rho}_{\lambda_3}(k_3) \epsilon^{*\sigma}_{\lambda_4}(k_4) \;\; ,
\end{aligned} \tag{3.2}$$

where

$$A \;=\; \alpha_N \; \alpha_N' \; \alpha_N'' \cdot \frac{g^4}{8 \cos^2 \theta_W} \;\; . \tag{3.3}$$

Using the narrow-width approximation for the N_1, N_2 propagators, and specializing to the case $m_1 = m_2 = m_N$, we obtain the following expression for the squared matrix element (summed over final and averaged over initial spins), the subscript in $\mathcal{M}_\pm$ denoting $\eta_{CP} = \pm 1$ ($q = p_1 + p_2$, $l = p_1 - p_2$):

$$\begin{aligned}
\overline{|\mathcal{M}_\pm|}^2 \quad = \quad & \frac{|A|^2}{2} \, \frac{1}{(s-m_Z^2)^2} \, \frac{\pi}{m_N \Gamma_n} \delta(q_1^2 - m_N^2) \, \frac{\pi}{m_N \Gamma_n} \delta(q_2^2 - m_N^2) \, \frac{m_N^2}{m_W^4} \cdot \\
\Big\{ \quad & f_1 \cdot \Big(\; \mp (m_N^2 - m_W^2)^2 (m_N^2 + 2m_W^2)^2 \cdot s \; \mp 4 (m_N^2 - 2m_W^2)^2 \cdot \\
& \Big[\; s \; (k_1 k_2)(q_1 q_2) - s \; (k_1 q_2)(k_2 q_1) - (k_1 k_2)(q_1 q)(q_2 q) + (k_1 k_2)(q_1 l)(q_2 l) \\
& + (k_1 q_2)(k_2 q)(q_1 q) - (k_1 q_2)(k_2 l)(q_1 l) + (k_1 q)(k_2 q_1)(q_2 q) - (k_1 l)(k_2 q_1)(q_2 l) \\
& - (k_1 q)(k_2 q)(q_1 q_2) + (k_1 l)(k_2 l)(q_1 q_2) \pm m_N^2 \; ((k_1 q)(k_2 q) - (k_1 l)(k_2 l)) \; \Big] \\
& + 2 \; (m_N^2 - m_W^2)(m_N^2 - 2m_W^2)^2 \; \Big[\; (k_1 q)(q_2 q) - (k_1 l)(q_2 l) + (k_2 q)(q_1 q) - (k_2 l)(q_1 l) \; \Big] \\
& + 8 \; m_W^2 (m_N^2 - m_W^2)^2 \; \Big[\; (q_1 q)(q_2 q) - (q_1 l)(q_2 l) \; \Big] \; \Big) \\
& -2 f_2 \cdot (m_N^2 - m_W^2)(m_N^4 - 4m_W^4) \; \Big(\; \pm (k_1 q)(q_1 l) - (k_1 q)(q_2 l) \\
& \mp (k_1 l)(q_1 q) + (k_1 l)(q_2 q) \pm (k_2 q)(q_2 l) - (k_2 q)(q_1 l) \\
& \mp (k_2 l)(q_2 q) + (k_2 l)(q_1 q) \; \Big) \quad \Big\} \; .
\end{aligned} \tag{3.4}$$

If the final state is $e^+ e^+$ instead of $e^- e^-$, we replace $f_2 \to -f_2$ in the above equation.

The expression for $\overline{|\mathcal{M}_\pm|^2}$ can be integrated over the phase space of W^+ and W^+ (i.e. over the momenta $k_3(= q_1 - k_1)$ and $k_4(= q_2 - k_2)$, in order to obtain the spectra in the lepton variables k_1 and k_2. Defining the four-vectors k_1 and k_2 in the e^+e^- c.m. frame by

$$k_1^\mu = E_1\ (1, \sin\theta_1\cos\phi_1, \sin\theta_1\sin\phi_1, \cos\theta_1)\ \ ,$$
$$k_2^\mu = E_2\ (1, \sin\theta_2\cos\phi_2, \sin\theta_2\sin\phi_2, \cos\theta_2)\ \ , \tag{3.5}$$

we have been able to derive the correlated distribution of the energies E_1 and E_2, as well as the correlation of the variables $\cos\theta_1$ and $\cos\theta_2$ measured relative to the e^- beam direction.

A. Energy Correlation

The normalized spectrum in the energies of the dilepton pair e^-e^- is ($\mathcal{E}_{1,2} = E_{1,2}/m_N$)

$$\frac{1}{\sigma}\cdot\left(\frac{d\sigma}{d\mathcal{E}_1 d\mathcal{E}_2}\right) \ = \ \mathcal{N}\ [a + b(\mathcal{E}_1+\mathcal{E}_2) + c(\mathcal{E}_1+\mathcal{E}_2)^2 - c(\mathcal{E}_1-\mathcal{E}_2)^2]\ , \tag{3.6}$$

where $\mathcal{N}$ is a normalization factor,

$$\mathcal{N} = [\ \mathcal{W}^2\beta^2(a + b\cdot\mathcal{W} + c\cdot\mathcal{W}^2)\ ]^{-1}\ \ , \tag{3.7}$$

with $\mathcal{W} = \sqrt{s}\cdot(m_N^2 - m_W^2)/2m_N^3$, $\beta = (1 - 4m_N^2/s)^{1/2}$. The coefficients a, b, c depend on the relative CP of the N_1N_2 system, and take the values

$$\begin{aligned}
a^+ &= m_N^2 m_W^2 (m_N^2 - m_W^2)^2 \left(2\ s - \frac{(m_N^2 + 2m_W^2)^2}{2m_N^2}\right)\ \ , \\
b^+ &= \sqrt{s}\ m_N^3 (m_N^2 - 2m_W^2)^2 (m_N^2 - m_W^2)\ \ , \qquad\qquad (\eta_{CP} = +1) \\
c^+ &= -m_N^6 (m_N^2 - 2m_W^2)^2\ \ ,
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
a^- &= 2\ m_W^2 (m_N^2 - m_W^2)^2 \left(s(s - 2m_N^2) - \frac{m_N^2}{m_W^2}\ (m_N^2 + 2m_W^2)^2\right)\ \ , \\
b^- &= \sqrt{s}\ m_N (m_N^2 - 2m_W^2)^2 (m_N^2 - m_W^2)(s - 2m_N^2)\ \ , \qquad (\eta_{CP} = -1) \\
c^- &= -m_N^4 (m_N^2 - 2m_W^2)^2 (s - 2m_N^2)\ \ .
\end{aligned} \tag{3.9}$$

Notice that the ratios b^+/a^+ and b^-/a^- are unequal (likewise the ratios c^+/a^+ and c^-/a^-), although $b^+/c^+ = b^-/c^-$. Thus the energy correlation of the two electrons in the final state is

different for the cases $\eta_{CP} = \pm 1$. This is illustrated in Fig. 3 for the hypothetical parameters $m_N = 500$ GeV, $\sqrt{s} = 1200$ GeV. It may be noted that the factor $f_2 = 4c_V c_A$ does not appear in the spectrum $(d\sigma/d\mathcal{E}_1 d\mathcal{E}_2)$, so that the energy correlation of e^+e^+ dileptons is the same as that of e^-e^-. In the limit $\beta \to 1$ the term $a^\pm$ dominates and the '+' and '−' cases are no more distinguishable.

B. Angular Correlation

Eq. (3.4) also allows a calculation of the correlated angular distribution of the final state e^-e^- system. Defining the angles $\theta_{1,2}$ as in Eq. (3.5), and integrating over all other variables, we find

$$
\begin{aligned}
&(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^\pm \ \sim \ \beta \cdot \int d\cos\theta_n \Big\{ f_1 \cdot \Big[\mp(m_N^2 + 2m_W^2)^2 \ s \cdot \mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2} \\
&+ (m_N^2 - 2m_W^2)^2 \ s \cdot \Big(\pm\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_n\beta - \cos\theta_1)(cos\theta_n\beta + cos\theta_2) \\
&+ \mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}(1 + \beta\cos\theta_n\cos\theta_1) + \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(1 - \beta\cos\theta_n\cos\theta_2) \Big) \\
&+ 2m_W^2 \ s^2 \cdot \mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2} \ (1 + \cos^2\theta_n\beta^2) \ - \ 4m_N^2(m_N^2 - 2m_W^2)^2 \\
&\cdot\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1 - \cos\theta_1\cos\theta_2) \Big] \\
&+ f_2 \cdot 2(m_N^4 - 4m_W^4) \ s \cdot \begin{bmatrix} \cos\theta_n\beta \ (\mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2} - \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}) & : & + \\ \mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}\cos\theta_1 + \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}\cos\theta_2 & : & - \end{bmatrix} \Big\} \ ,
\end{aligned} \tag{3.10}
$$

with

$$
\mathcal{K}_1^{\theta_{1(2)}} = \frac{2A_{1(2)}}{(A_{1(2)}^2 - B_{1(2)}^2)^{3/2}} \ , \ \mathcal{K}_2^{\theta_{1(2)}} = \frac{2A_{1(2)}^2 + B_{1(2)}^2}{(A_{1(2)}^2 - B_{1(2)}^2)^{5/2}} \ , \tag{3.11}
$$

$$
A_{1(2)} = 1 - (+)\beta\cos\theta_n\cos\theta_{1(2)} \ , \ B_{1(2)} = \beta\sin\theta_n\sin\theta_{1(2)} \ .
$$

The correlation (3.10) has been evaluated for $m_N = 500$ GeV and $\sqrt{s} = 1200$ GeV (using $f_1 = 1 + 4\sin^2\theta_W + 8\sin^4\theta_4$, $f_2 = 1 - 4\sin^2\theta_W$) and is plotted in Fig. 4. There is a clear difference between the cases $\eta_{CP} = \pm 1$. The angular correlation in (3.10) becomes particularly transparent near the threshold $\beta \to 0$, where we obtain the analytic results

$$\frac{1}{\sigma^+} \cdot (\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^+_{\beta\to 0} \approx \frac{1}{4} \cdot [\ 1 + \frac{1}{2} \cdot \frac{f_2}{f_1} \cdot \frac{m_N^2 - 2m_W^2}{m_N^2 + 2m_W^2} \cdot (\cos\theta_1 + \cos\theta_2)\ ]\ , \tag{3.12}$$

$$\begin{aligned}\frac{1}{\sigma^-} \cdot (\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^-_{\beta\to 0} \approx \frac{1}{4} \cdot [\ 1 &+ \frac{(m_N^2 - 2m_W^2)^2}{(m_N^2 + 2m_W^2)^2} \cdot \cos\theta_1 \cos\theta_2 \\ &+ \frac{f_2}{f_1} \cdot \frac{(m_N^2 - 2m_W^2)}{(m_N^2 + 2m_W^2)} \cdot (\cos\theta_1 + \cos\theta_2)\ ]\ .\end{aligned} \tag{3.13}$$

Notice that the distribution in the variables $\cos\theta_1$ and $\cos\theta_2$ becomes flat in the case $\eta_{CP} = +1$ when f_2/f_1 is neglected. By contrast, there remains a nontrivial correlation for $\eta_{CP} = -1$, even in the absence of f_2. As before, the above results for e^-e^- hold for e^+e^+ if one replaces $f_2 \to -f_2$.

IV. UNLIKE-SIGN DILEPTONS: THE REACTION $e^+e^- \to N_1 N_2 \to W^+W^-e^+e^-$

Proceeding as in Sec. 3, the matrix element for the reaction $e^+e^- \to N_1N_2 \to W^+W^-e^+e^-$ (Fig. 2) is

$$\begin{aligned}\mathcal{M}_m = iA\ \ j^e_\mu\ \ \Delta_Z^{\mu\nu} &\frac{1}{q_1^2 - m_1^2 + im_1\Gamma_1} \cdot \frac{1}{q_2^2 - m_2^2 + im_2\Gamma_2} \cdot \\ [\ \ &\lambda_2\ \bar{u}_{t_1}(k_1)\ \gamma_\rho\ \not{q}_1\ \gamma_\nu\ \not{q}_2\ \gamma_\sigma\ \frac{1}{2}(1-\gamma_5)\ v_{t_2}(k_2) \\ + &\lambda_1\ m_1 m_2\ \bar{u}_{t_1}(k_1)\ \gamma_\rho\ \gamma_\nu\ \gamma_\sigma\ \frac{1}{2}(1-\gamma_5)\ v_{t_2}(k_2)\ ]\ \epsilon^{*\rho}_{\lambda_3}(k_3)\epsilon^{*\sigma}_{\lambda_4}(k_4)\ \ .\end{aligned} \tag{4.1}$$

The same final state, produced via a Dirac particle-antiparticle pair ($e^+e^- \to N\bar{N} \to W^+W^-e^+e^-$), has the amplitude

$$\begin{aligned}\mathcal{M}_d = iA\ \ j^e_\mu\ \ \Delta_Z^{\mu\nu} &\frac{1}{q_1^2 - m_1^2 + im_1\Gamma_1} \cdot \frac{1}{q_2^2 - m_2^2 + im_2\Gamma_2} \cdot \\ &\bar{u}_{t_1}(k_1)\ \gamma_\rho\ \not{q}_1\ \gamma_\nu\ \not{q}_2\ \gamma_\sigma\ \frac{1}{2}(1-\gamma_5)\ v_{t_2}(q_2)\ \epsilon^{*\rho}_{\lambda_3}(k_3)\epsilon^{*\sigma}_{\lambda_4}(k_4)\ \ .\end{aligned} \tag{4.2}$$

Summing (averaging) over final (initial) polarizations, and using the narrow-width approximation for the N_1, N_2 propagators, we obain the squared matrix elements given below:

$$\overline{|\mathcal{M}_\pm|}^2 \quad = \quad \frac{|A|^2}{2}\ \frac{1}{(s-m_Z^2)^2}\ \frac{\pi}{m_N\Gamma_n}\delta(q_1^2 - m_N^2)\ \frac{\pi}{m_N\Gamma_n}\delta(q_2^2 - m_N^2)\ \frac{m_N^2}{m_W^4} \cdot$$

$$\Big\{ \quad f_1 \cdot \Big(\ \mp (m_N^2 - m_W^2)^2 (m_N^2 + 2m_W^2)^2 \cdot s \ \pm 4(m_N^2 - 2m_W^2)^2 \cdot$$

$$\Big[\, s \, (k_1k_2)(q_1q_2) - s \, (k_1q_2)(k_2q_1) - (k_1k_2)(q_1q)(q_2q) + (k_1k_2)(q_1l)(q_2l)$$

$$+ \, (k_1q_2)(k_2q)(q_1q) - (k_1q_2)(k_2l)(q_1l) + (k_1q)(k_2q_1)(q_2q) - (k_1l)(k_2q_1)(q_2l)$$

$$- \, (k_1q)(k_2q)(q_1q_2) + (k_1l)(k_2l)(q_1q_2) \pm m_N^2 \, ((k_1q)(k_2q) - (k_1l)(k_2l)) \, \Big]$$

$$- \, 2 \, (m_N^2 - m_W^2)(m_N^2 - 2m_W^2)^2 \, \Big[\, (k_1q)(q_2q) - (k_1l)(q_2l) + (k_2q)(q_1q) - (k_2l)(q_1l) \, \Big]$$

$$+ \, 2 \, (m_N^2 + 4m_W^2/m_N^2)(m_N^2 - m_W^2)^2 \, [\, (q_1q)(q_2q) - (q_1l)(q_2l) \,] \ \Big)$$

$$-2f_2 \cdot \Big(\, (m_N^2 - m_W^2)(m_N^4 - 4m_W^4) \, (\ \pm (k_1q)(q_1l) - (k_1q)(q_2l)$$

$$\mp (k_1l)(q_1q) + (k_1l)(q_2q) \mp (k_2q)(q_2l) + (k_2q)(q_1l)$$

$$\pm (k_2l)(q_2q) - (k_2l)(q_1q) \ \ )$$

$$+2(m_N^2 + 4m_W^4/m_N^2)(m_N^2 - m_W^2)^2 \, \Big[(q_1q)(q_2l) - (q_2q)(q_1l) \Big] \Big) \quad \Big\} \ , \tag{4.3}$$

$$\overline{|\mathcal{M}_d|}^2 \quad = \quad |A|^2 \, \frac{1}{(s - m_Z^2)^2} \, \frac{\pi}{m_N \Gamma_n} \delta(q_1^2 - m_N^2) \, \frac{\pi}{m_N \Gamma_n} \delta(q_2^2 - m_N^2)$$

$$\Big\{ \, f_1 \cdot \Big(\ \frac{m_N^4}{m_W^4} (m_N^2 - 2m_W^2)^2 \cdot \Big[(k_1q)(k_2q) - (k_1l)(k_2l) \Big]$$

$$+2 \frac{m_N^2}{m_W^2} (m_N^2 - m_W^2)(m_N^2 - 2m_W^2) \cdot \Big[(k_1q)(q_2q) - (k_1l)(q_2l) + (k_2q)(q_1q) - (k_2l)(q_1l) \Big]$$

$$+4(m_N^2 - m_W^2)^2 \cdot \Big[(q_1q)(q_2q) - (q_1l)(q_2l) \Big] \ \Big)$$

$$+ \, f_2 \cdot \Big(\ \frac{m_N^4}{m_W^4} (m_N^2 - 2m_W^2)^2 \cdot \Big[(q_1q)(k_2l) - (k_2q)(k_1l) \Big]$$

$$+2 \frac{m_N^2}{m_W^2} (m_N^2 - m_W^2)(m_N^2 - 2m_W^2) \cdot \Big[(k_1q)(q_2l) - (k_2q)(q_1l) + (q_1q)(k_2l) - (q_2q)(k_1l) \Big]$$

$$+4(m_N^2 - m_W^2)^2 \cdot \Big[(q_1q)(q_2l) - (q_2q)(q_1l) \Big] \ \Big) \quad \Big\} \ . \tag{4.4}$$

In complete analogy with the discussion of like-sign leptons (Sec. 3), we derive from the above equations the correlation in the energies and angles of the final e^+e^- state.

A. Energy Correlation

The distribution in the scaled energies $\mathcal{E}_1$, $\mathcal{E}_2$ has the quadratic form given in Eq. (3.6), where the coefficients in the Majorana cases '+' and '−' and the Dirac case 'd' now have

the values

$$
\begin{aligned}
a^{+} &= \frac{1}{2} m_N^2 (m_N^2 - m_W^2)^2 \Big(\frac{s}{m_N^2} (m_N^4 + 4m_W^4) - 2(m_N^2 + 2m_W^2)^2 \Big) \ , \\
b^{+} &= -\sqrt{s}\, m_N^3 (m_N^2 - 2m_W^2)^2 (m_N^2 - m_W^2) \ , \\
c^{+} &= m_N^6 (m_N^2 - 2m_W^2)^2 \ , \\
a^{-} &= \frac{1}{2} m_N^2 (m_N^2 - m_W^2)^2 \Big(s(s - 2m_N^2)(1 + 4\frac{m_W^4}{m_N^4}) - 4(m_N^2 + 2m_W^2)^2 \Big) \ , \\
b^{-} &= -\sqrt{s}\, m_N (m_N^2 - 2m_W^2)^2 (m_N^2 - m_W^2)(s - 2m_N^2) \ , \\
c^{-} &= m_N^4 (m_N^2 - 2m_W^2)^2 (s - 2m_N^2) \ , \\
a^{d} &= (m_N^2 - m_W^2)^2 \Big(4(s - m_N^2)(s - 4m_N^2) \frac{m_W^4}{m_N^2} \\
&\quad - (m_N^2 - 2m_W^2) \big(2(s - 4m_N^2) m_W^2 - m_N^2 (m_N^2 - 2m_W^2) \big) \Big) \ , \\
b^{d} &= \sqrt{s}\, m_N (m_N^2 - 2m_W^2)(m_N^2 - m_W^2) \Big(4 s m_W^2 - (m_N^2 + 14 m_W^2) m_N^2 \Big) \ , \\
c^{d} &= m_N^4 (m_N^2 - 2m_W^2)^2 (s - 3m_N^2) \ .
\end{aligned} \tag{4.5}
$$

The corresponding three distributions are plotted in Fig. 5. As in the case of like-sign dileptons, the e^+e^- pairs have distinct correlations for $\eta_{CP} = \pm 1$. A comparison of the Majorana cases with the Dirac case reveals an interesting difference. In the Majorana cases the total e^+e^- energy $Y = \mathcal{E}_1 + \mathcal{E}_2$ is distributed symmetrically around the mid-point of this variable $Y_0 = 1/2(Y_{min} + Y_{max})$. By contrast, e^+e^- pairs resulting from Dirac $N\bar{N}$ primary state have a total energy distribution that is unsymmetric around the mid-point.

B. Angle Correlation

In analogy to the distribution $d\sigma/d\cos\theta_1 d\cos\theta_2$ obtained for e^-e^- pairs (Eq. 3.10), the result for unlike-sign dileptons e^+e^- is

$$
\begin{aligned}
&(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^{\pm} \ \sim \ \beta \cdot \int \ d\cos\theta_n \ \Big\{ \ f_1 \cdot \\
&\Big[\mp 2m_N^2 (m_N^2 + 2m_W^2)^2 \ s \cdot \mathcal{K}_1^{\theta_1} \mathcal{K}_1^{\theta_2} \\
&- 2m_N^2 (m_N^2 - 2m_W^2)^2 \ s \cdot \Big(\ \pm \mathcal{K}_2^{\theta_1} \mathcal{K}_2^{\theta_2} (\cos\theta_n \beta - \cos\theta_1)(cos\theta_n \beta + cos\theta_2)
\end{aligned}
$$

$$+ \mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}(1+\beta\cos\theta_n\cos\theta_1) + \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\beta\cos\theta_n\cos\theta_2)\Big)$$

$$+ (m_N^4 + 4m_W^4)\; s^2 \cdot \mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2}\;(1+\cos^2\theta_n\beta^2) \;+\; 8m_N^4(m_N^2-2m_W^2)^2 \cdot$$

$$\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\cos\theta_1\cos\theta_2)\Big]$$

$$+f_2 \cdot 2(m_N^4 - 4m_W^4)\; s \cdot \Big[-\mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2} \cdot s\;\cos\theta_n\beta$$

$$+2m_N^2 \cdot \begin{bmatrix} \cos\theta_n\beta\;(\mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2} + \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}) & : & + \\ \mathcal{K}_2^{\theta_1}\mathcal{K}_1^{\theta_2}\cos\theta_1 - \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}\cos\theta_2 & : & - \end{bmatrix} \Big] \Big\} \;, \tag{4.6}$$

$$(\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^d \;\sim\; \beta \cdot \int d\cos\theta_n \Big\{ f_1 \cdot$$

$$\Big[s^2 m_W^4 \cdot \mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2} \cdot (1+\cos\theta_n\beta^2)$$

$$+ m_N^2 m_W^2(m_N^2 - 2m_W^2)\; s \cdot \Big(\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(1+\beta\cos\theta_n\cos\theta_1) + \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(1-\beta\cos\theta_n\cos\theta_2)\Big)$$

$$+ m_N^4(m_N^2-2m_W^2)^2 \cdot \mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2} \cdot (1-\cos\theta_1\cos\theta_2)\Big]$$

$$+f_2 \cdot \Big[2s^2 m_W^4 \cdot \mathcal{K}_1^{\theta_1}\mathcal{K}_1^{\theta_2} \cdot \cos\theta_n\beta$$

$$+ m_N^2 m_W^2(m_N^2 - 2m_W^2)\; s \cdot \Big(\mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_n\beta + \cos\theta_1) + \mathcal{K}_1^{\theta_1}\mathcal{K}_2^{\theta_2}(\cos\theta_n\beta - \cos\theta_2)\Big)$$

$$+ m_N^4(m_N^2-2m_W^2)^2 \cdot \mathcal{K}_2^{\theta_1}\mathcal{K}_2^{\theta_2} \cdot (\cos\theta_1 - \cos\theta_2)\Big]\Big\}\;. \tag{4.7}$$

As usual, the indices '+', '−' and 'd' differentiate between the Majorana cases $\eta_{CP} = +1$, -1 and the Dirac case. The angle-correlations expressed by Eqs. (4.6), (4.7) are plotted in Fig. 6., where the differences between the three cases are obvious. Close to threshold ($\beta \to 0$), the correlation between $\cos\theta_1$ and $\cos\theta_2$ can be presented in analytic form

$$\frac{1}{\sigma^+} \cdot (\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^+_{\beta\to 0} \approx \frac{1}{4} \cdot [\,1 + \frac{1}{2} \cdot \frac{f_2}{f_1} \cdot \frac{m_N^2 - 2m_W^2}{m_N^2 + 2m_W^2} \cdot (\cos\theta_1 - \cos\theta_2)\,]\;, \tag{4.8}$$

$$\frac{1}{\sigma^d} \cdot (\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^d_{\beta\to 0} = \frac{1}{\sigma^-} \cdot (\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2})^-_{\beta\to 0} \tag{4.9}$$

$$\approx \frac{1}{4} \cdot [\,1 - \frac{(m_N^2-2m_W^2)^2}{(m_N^2+2m_W^2)^2} \cdot \cos\theta_1\cos\theta_2 \;+\; \frac{f_2}{f_1} \cdot \frac{(m_N^2-2m_W^2)}{(m_N^2+2m_W^2)} \cdot (\cos\theta_1 - \cos\theta_2)\,]\;.$$

In this limit, the cases '+' and '−' remain distinct, but the case $\eta_{CP} = -1$ converges to the Dirac case.

V. COMMENTS

We comment briefly on some other papers which have a partial overlap with the considerations presented above.

(i) Our discussion of the production reaction $e^+e^- \to N_1 N_2$ follows very closely that given in Ref. [1]. Our results for $d\sigma/d\Omega$ given in the Appendix (Eqs. (A.1)–(A.6)) essentially coincide with those in this paper, with two minor differences: The angular distributions for the case of two distinct Majorana particles with the same CP-parity, as well as for the case of two distinct Dirac particles (Eq. (4E) and (4D) in Ref. [1]), are slightly different from our distributions, presented in the Appendix (Eq. (A.2) and (A.3)).

(ii) The cross section for the Majorana process $e^+e^- \to N_1 N_2$, with $m_1 = m_2$ and $\eta_{CP} = +1$ calculated in Ref. [2] agrees with that obtained in this paper. However the Dirac case $e^+e^- \to N\bar{N}$ (Eq. (2) of Ref. [2]) differs from our result (Eq. (A.5)), as also noted in Ref. [1].

(iii) The spin-summed differential cross section for the Majorana process $e^+e^- \to N_1 N_2$ (with $m_1 = m_2$, $\eta_{CP} = +1$) calculated in the present paper, as well as in Refs. [1,2], differs from that given in Ref. [4], but agrees with the results given in Refs. [3,6,10].

(iv) Our analysis of heavy Majorana production and decay has been essentially model-independent. Discussions in the context of specific gauge models, based on $SU(2)_L \times SU(2)_R \times U(1)$ or $E(6)$ symmetries, may be found in Refs. [6,10,11].

APPENDIX A: DIFFERENTIAL CROSS SECTION FOR $e^+e^- \to N_1 N_2$

Following Ref. [1], we consider the following five cases, in which N_1 and N_2 are

A distinct Dirac particles.

B distinct Majorana particles with the same CP-parity.

C distinct Majorana particles with opposite CP-parity.

D Dirac particle-antiparticle pair.

E identical Majorana particles.

Choosing the N_1 direction in the e^+e^- c.m. system to be the z-axis, and the e^--beam direction to be at an angle θ (= scattering angle), the momenta (q_1, q_2) and spins (t_1, t_2) of N_1 and N_2 have components

$$\begin{aligned} N_1 \ : \ q_1^\mu &= (\gamma m_1, 0, 0, \gamma\beta m_1) \ , \\ t_1^\mu &= (\gamma\beta n_z, n_x, n_y, \gamma n_z) \ , \\ N_2 \ : \ q_2^\mu &= (\gamma' m_2, 0, 0, -\gamma'\beta' m_2) \ , \\ t_2^\mu &= (-\gamma'\beta' n'_z, n'_x, n'_y, \gamma' n'_z) \ . \end{aligned} \tag{A.1}$$

The differential cross sections are (with $\beta = (1 - 4m_1^2/s)^{1/2}$, $\beta' = (1 - 4m_2^2/s)^{1/2}$, $\gamma = (1 - \beta^2)^{-1/2}$, $\gamma' = (1 - \beta'^2)^{-1/2}$, $\lambda(x, y, z) = [\ x^2 + y^2 + z^2 - 2(xy + yz + zx)\ ]^{1/2}$)

$$\begin{aligned} (\frac{d\sigma}{d\Omega})_A = & \frac{G_F^2 \alpha_N^2}{512\pi^2} \ |R(s)|^2 \ [1 - (m_1^2 - m_2^2)^2/s^2] \ \lambda(s, m_1^2, m_2^2) \\ & \Big\{ \ f_1 \ \ [\ (1 + C^2\beta'\beta) - (\beta n'_z + \beta' n_z)(1 + C^2) - (\beta' n_x/\gamma + \beta n'_x/\gamma')SC \\ & \qquad + n_z n'_z (C^2 + \beta\beta') + (n_x n'_z/\gamma + n'_x n_z/\gamma')SC + n_x n'_x S^2/\gamma\gamma' \ ] \\ & + f_2 \ \ [\ C(\beta + \beta') - (n_z + n'_z)C(1 + \beta\beta') - (n_x/\gamma + n'_x/\gamma')S \\ & + n_z n'_z C(\beta + \beta') + S(\beta' n_x n'_z/\gamma + \beta n'_x n_z/\gamma') \ ] \ \ \Big\} \ , \end{aligned} \tag{A.2}$$

$$
\begin{aligned}
(\frac{d\sigma}{d\Omega})_B = & \frac{G_F^2\alpha_N^2}{256\pi^2}\ |R(s)|^2\ [1-(m_1^2-m_2^2)^2/s^2]\ \lambda(s,m_1^2,m_2^2) \\
& \Big\{\ f_1\ \ [\ n_x n'_x S^2(1/\gamma\gamma'-1) + n_y n'_y S^2\beta\beta' + n_z n'_z(\beta\beta' - C^2(1/\gamma\gamma'-1)) \\
& \qquad + (n_x n'_z - n'_x n_z) SC(1/\gamma - 1/\gamma') +\ C^2\beta\beta' - 1/\gamma\gamma' + 1\ ] \\
& + f_2\ \ [\ (n_x - n'_x)S(1/\gamma' - 1/\gamma) + (n_z + n'_z)C(1/\gamma\gamma' - \beta\beta' - 1)\ ]\ \Big\}\ ,
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
(\frac{d\sigma}{d\Omega})_C = & \frac{G_F^2\alpha_N^2}{256\pi^2}\ |R(s)|^2\ [1-(m_1^2-m_2^2)^2/s^2]\ \lambda(s,m_1^2,m_2^2) \\
& \Big\{\ f_1\ \ [\ n_x n'_x S^2(1/\gamma\gamma'+1) - n_y n'_y S^2\beta\beta' + n_z n'_z(\beta\beta' + C^2(1/\gamma\gamma'+1)) \\
& \qquad + (n_x n'_z + n'_x n_z) SC(1/\gamma + 1/\gamma') +\ C^2\beta\beta' + 1/\gamma\gamma' + 1\ ] \\
& - f_2\ \ [\ (n_x + n'_x)S(1/\gamma + 1/\gamma') + (n_z + n'_z)C(1/\gamma\gamma' + \beta\beta' + 1)\ ]\ \Big\}\ ,
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
(\frac{d\sigma}{d\Omega})_D = (\frac{d\sigma}{d\Omega})_{A,m_1=m_2} \ \ = & \ \ \frac{G_F^2\alpha_N^2}{512\pi^2}\ |R(s)|^2\ \lambda(s,m^2,m^2) \\
& \Big\{\ f_1\ \ [(1+C^2\beta^2) - (n_z+n'_z)\beta(1+C^2) - (n_x+n'_x)SC\beta/\gamma + n_z n'_z(C^2+\beta^2) \\
& + (n_x n'_z + n_z n'_x)SC/\gamma + n_x n'_x S^2/\gamma^2] \ \ + \ \ f_2\ \ [\ 2C\beta - (n_z+n'_z)C(1+\beta^2) \\
& - (n_x+n'_x)S/\gamma + 2n_z n'_z C\beta + (n_x n'_z + n'_x n_z)S\beta/\gamma\ ]\ \Big\}\ ,
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
(\frac{d\sigma}{d\Omega})_E = \frac{1}{2}\cdot(\frac{d\sigma}{d\Omega})_{B,m_1=m_2} \ \ = & \ \ \frac{G_F^2\alpha_N^2}{512\pi^2}\ |R(s)|^2\ \lambda(s,m^2,m^2)\beta^2 \\
& \Big\{\ f_1\ \ [\ (n_y n'_y - n_x n'_x)S^2 + (1+n_z n'_z)(1+C^2)\ ]\ -\ f_2\ \ 2(n_z+n'_z)C\ \Big\}\ .
\end{aligned}
\tag{A.6}
$$

APPENDIX B: MATRIX ELEMENTS FOR $e^+e^- \to N_1N_2 \to e^\pm e^- W^\mp W^+$

1. Like Sign Dileptons

The matrix element for the reaction (Fig. 2)

$$
e^+(p_2,t_2) + e^-(p_1,t_1)\ \ \to\ \ e^-(k_1)e^-(k_2)W^+(k_3,\lambda_3)W^+(k_4,\lambda_4)
\tag{B.1}
$$

is

$$
\begin{aligned}
\mathcal{M} \;=\; iA\, j^e_\mu\, \Delta_Z^{\mu\nu} \Big\{ & \lambda_2\, \bar{u}_{t_1}(k_1)\, \gamma_\rho\, \frac{1}{2}(1-\gamma_5)\, \frac{\not{q}_1 + m_1}{q_1^2 - m_1^2 + i m_1 \Gamma_1}\, \gamma_\nu\, \frac{1}{2}(1-\gamma_5) \\
& \frac{-\not{q}_2 + m_2}{q_2^2 - m_2^2 + i m_2 \Gamma_2}\, \gamma_\sigma\, \frac{1}{2}(1+\gamma_5)\, v_{t_2}(k_2) \\
& - \lambda_1\, \bar{u}_{t_2}(k_2)\, \gamma_\sigma\, \frac{1}{2}(1-\gamma_5)\, \frac{\not{q}_2 + m_2}{q_2^2 - m_2^2 + i m_2 \Gamma_2}\, \gamma_\nu\, \frac{1}{2}(1-\gamma_5) \\
& \frac{-\not{q}_1 + m_1}{q_1^2 - m_1^2 + i m_1 \Gamma_1}\, \gamma_\rho\, \frac{1}{2}(1+\gamma_5)\, v_{t_1}(k_1) \Big\}\, \epsilon^{*\rho}_{\lambda_3}(k_3) \epsilon^{*\sigma}_{\lambda_4}(k_4) \quad . \qquad \text{(B.2)}
\end{aligned}
$$

Upon rearrangement, this gives the matrix element in Eq. (3.2).

2. Unlike Sign Dileptons

The matrix element for the Majorana-mediated process (Fig. 2)

$$
e^+(p_2, t_2) + e^-(p_1, t_1) \;\to\; e^-(k_1) e^-(k_2) W^+(k_3, \lambda_3) W^+(k_4, \lambda_4) \qquad \text{(B.3)}
$$

is

$$
\begin{aligned}
\mathcal{M}_m \;=\; -iA\, j^e_\mu\, \Delta_Z^{\mu\nu} \Big\{ & \lambda_2\, \bar{u}_{t_1}(k_1)\, \gamma_\rho\, \frac{1}{2}(1-\gamma_5)\, \frac{\not{q}_1 + m_1}{q_1^2 - m_1^2 + i m_1 \Gamma_1}\, \gamma_\nu\, \frac{1}{2}(1-\gamma_5) \\
& \frac{-\not{q}_2 + m_2}{q_2^2 - m_2^2 + i m_2 \Gamma_2}\, \gamma_\sigma\, \frac{1}{2}(1-\gamma_5)\, v_{t_2}(k_2) \\
& - \lambda_1\, \bar{u}_{t_1}(k_1)\, \gamma_\rho\, \frac{1}{2}(1-\gamma_5)\, \frac{\not{q}_1 + m_1}{q_1^2 - m_1^2 + i m_1 \Gamma_1}\, \gamma_\nu\, \frac{1}{2}(1+\gamma_5) \\
& \frac{-\not{q}_2 + m_2}{q_2^2 - m_2^2 + i m_2 \Gamma_2}\, \gamma_\sigma\, \frac{1}{2}(1-\gamma_5)\, v_{t_2}(k_2) \Big\}\, \epsilon^{*\rho}_{\lambda_3}(k_3) \epsilon^{*\sigma}_{\lambda_4}(k_4) \quad . \qquad \text{(B.4)}
\end{aligned}
$$

Upon rearrangement, this gives the matrix element in Eq. (4.1).

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FIGURE CAPTIONS

1. Feynman diagram for the reaction $e^+e^- \to N_1N_2$

2. Diagram showing the sequential process $e^+e^- \to N_1N_2 \to e^\pm e^- W^\mp W^+$

3. Energy correlation of the e^-e^- lepton pair in the reaction $e^+e^- \to N_1N_2 \to e^-W^+e^-W^+$, for the cases (a) $\eta_{CP} = +1$, (b) $\eta_{CP} = -1$. (Paramers for this and succeeding figures: $\sqrt{s} = 1.2$ TeV, $m_N = 500$ GeV.)

4. Angle correlation of e^-e^- dileptons in $e^+e^- \to N_1N_2 \to e^-W^+e^-W^+$, for (a) $\eta_{CP} = +1$, (b) $\eta_{CP} = -1$

5. Energy correlation of e^-e^+ dileptons in $e^+e^- \to N_1N_2 \to e^-W^+e^+W^-$: (a) Majorana pair, $\eta_{CP} = +1$, (b) Majorana pair, $\eta_{CP} = -1$, (c) Dirac $N\bar{N}$-pair

6. Angle correlation of e^-e^+ dileptons in $e^+e^- \to N_1N_2 \to e^-W^+e^+W^-$: (a) Majorana pair, $\eta_{CP} = +1$, (b) Majorana pair, $\eta_{CP} = -1$, (c) Dirac $N\bar{N}$-pair

FIGURES

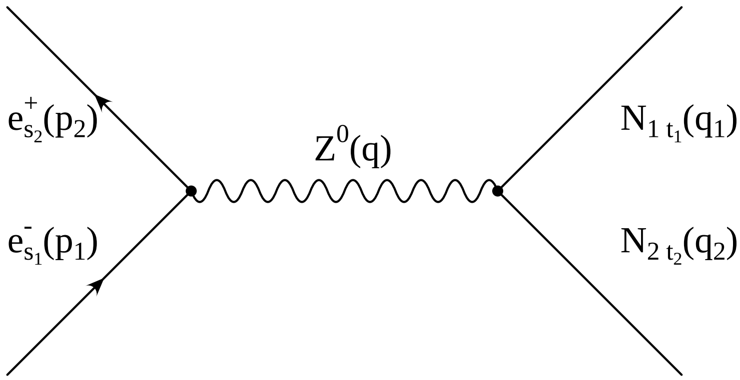


FIG. 1.

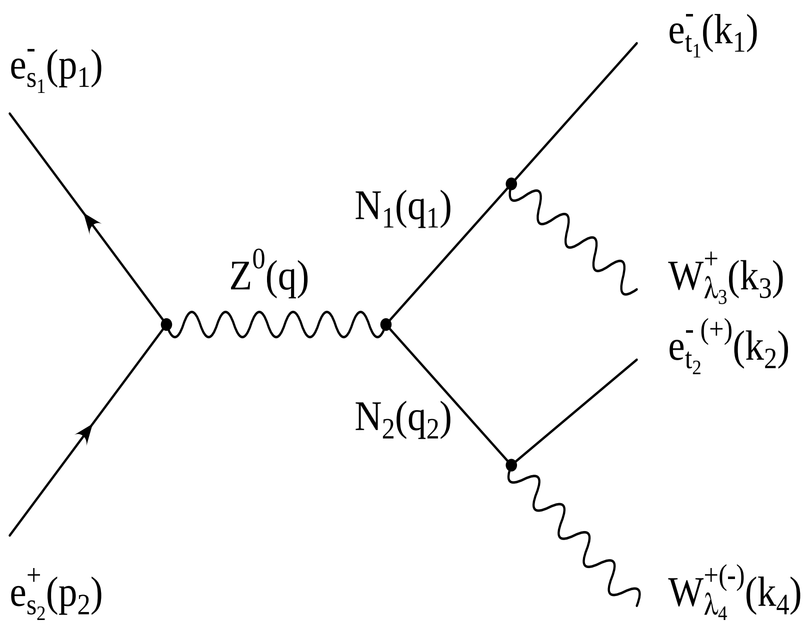
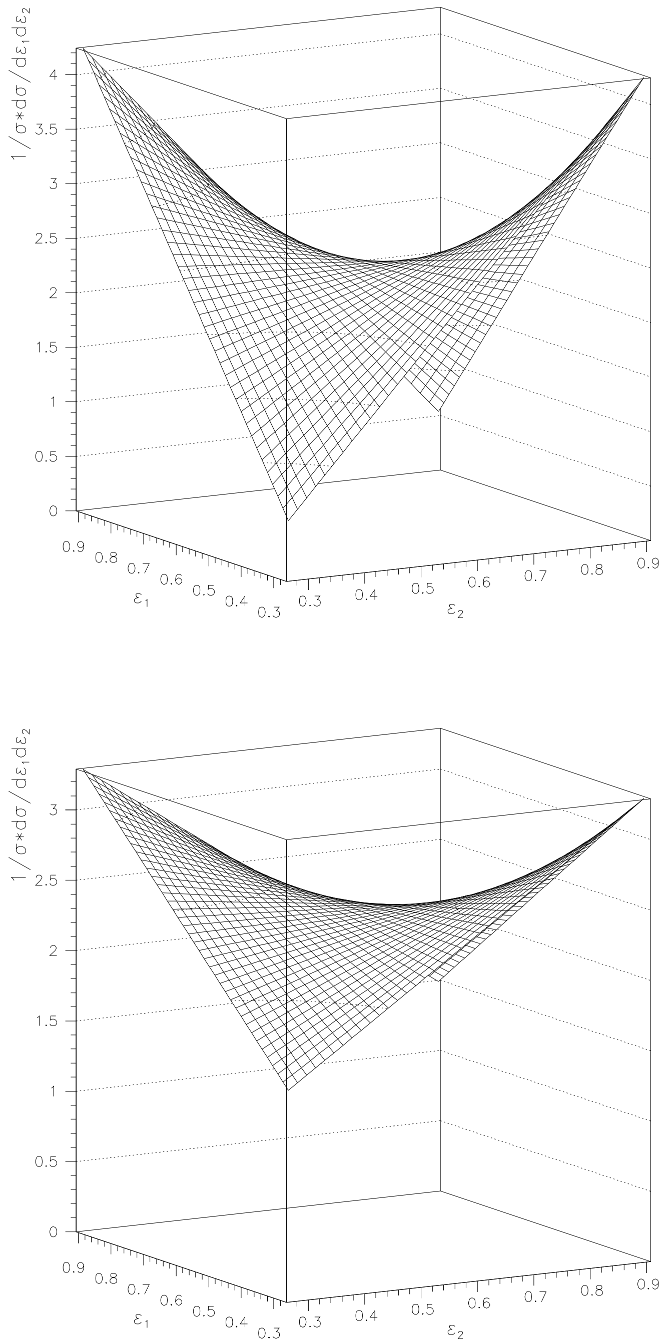


FIG. 2.

e^-e^- Final State: Energy Correlation



(a) Majorana (+)

(b) Majorana (−)

FIG. 3.

$e^{-}e^{-}$ Final State: Angular Correlation

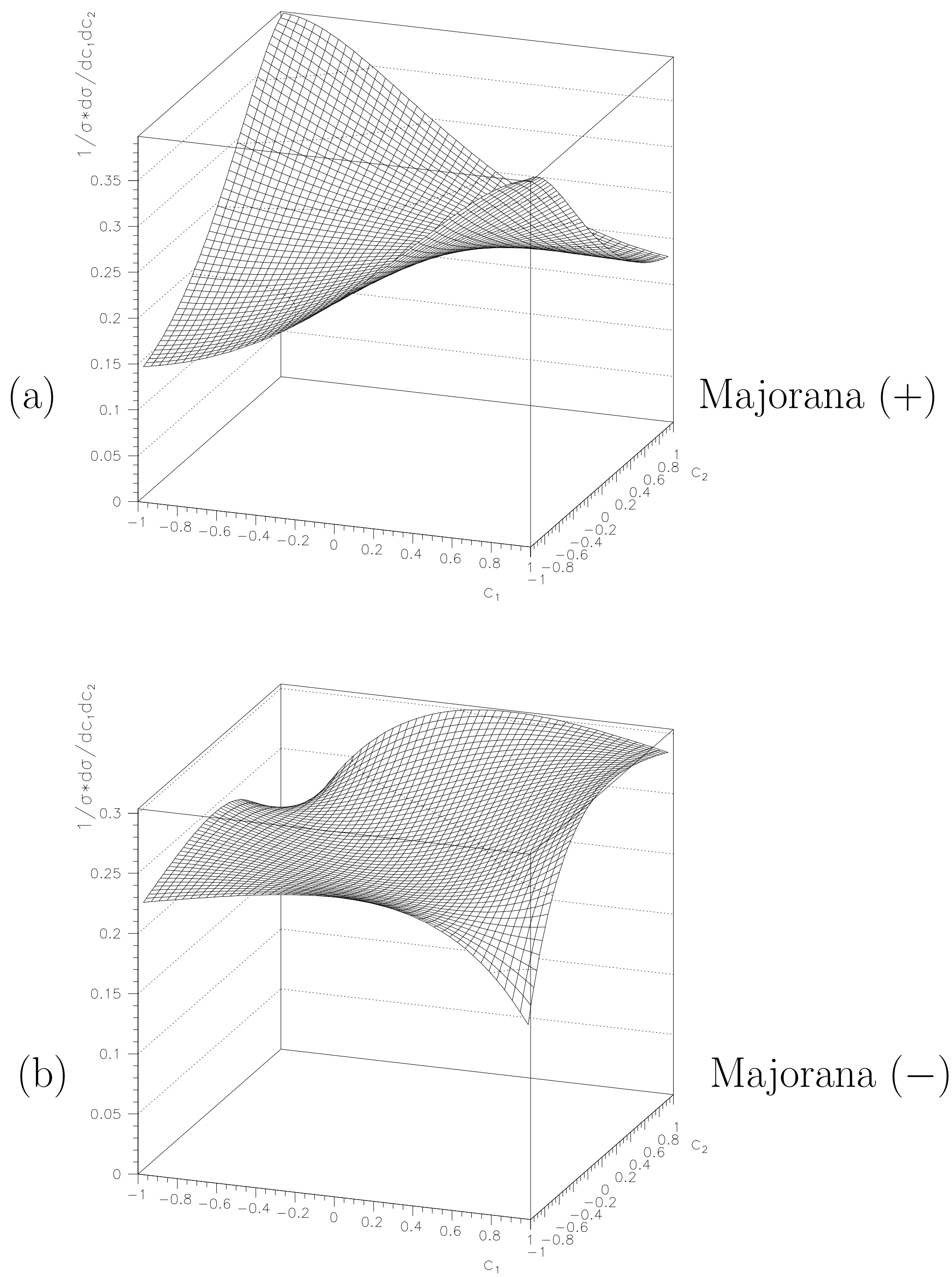


FIG. 4.

e^+e^- Final State: Energy Correlation

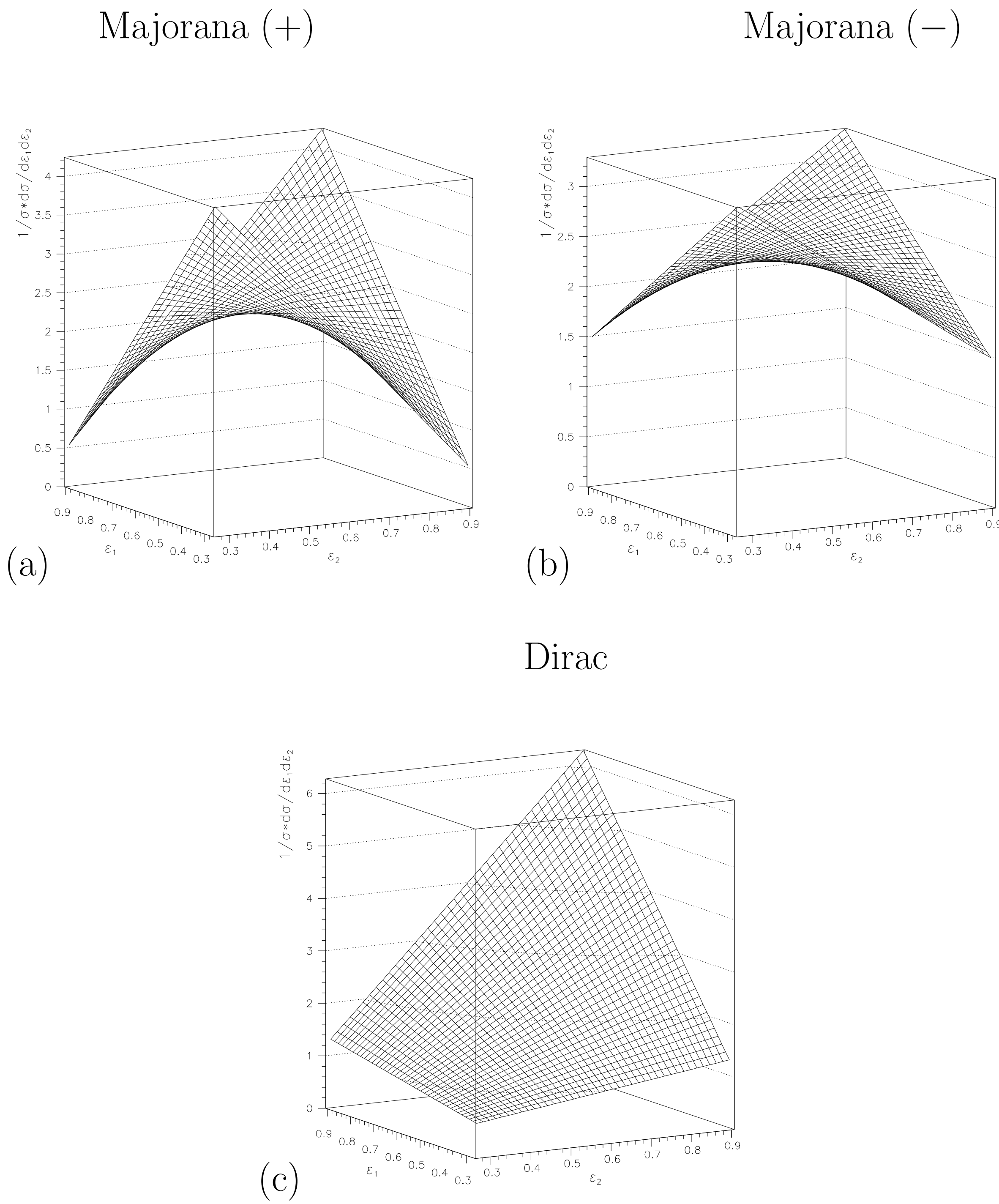
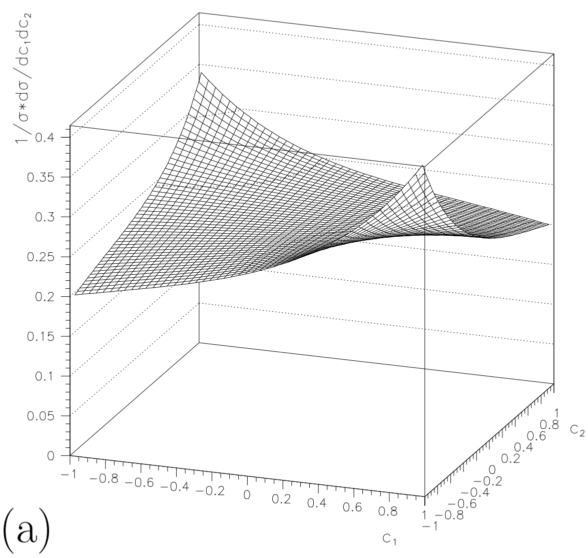


FIG. 5.

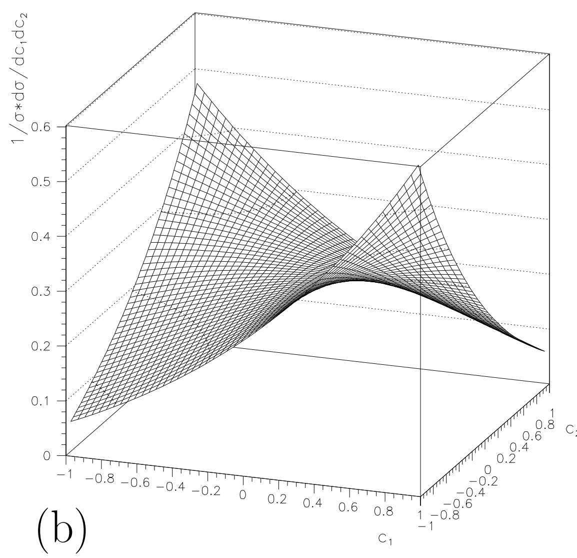
e^+e^- Final State: Angular Correlation

Majorana (+)



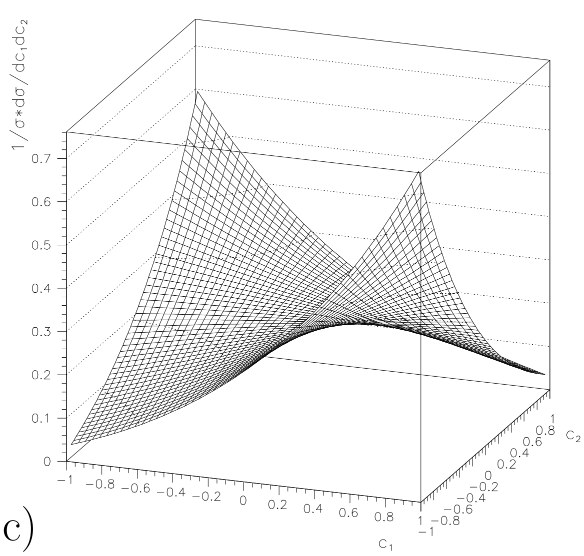
(a)

Majorana (−)



(b)

Dirac



(c)

FIG. 6.